



## AP<sup>®</sup> Calculus BC 2013 Free-Response Questions

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**2013 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

**CALCULUS BC  
SECTION II, Part A**

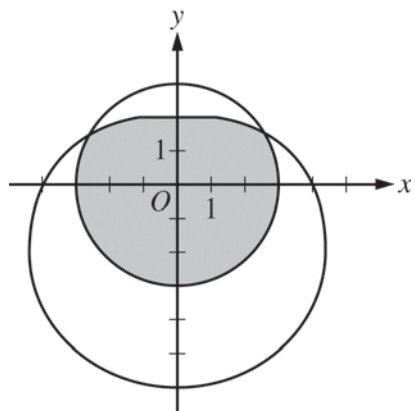
**Time—30 minutes**

**Number of problems—2**

**A graphing calculator is required for these problems.**

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by  $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$ , where  $t$  is measured in hours and  $0 \leq t \leq 8$ . At the beginning of the workday ( $t = 0$ ), the plant has 500 tons of unprocessed gravel. During the hours of operation,  $0 \leq t \leq 8$ , the plant processes gravel at a constant rate of 100 tons per hour.
- (a) Find  $G'(5)$ . Using correct units, interpret your answer in the context of the problem.
- (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
- (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time  $t = 5$  hours? Show the work that leads to your answer.
- (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.
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2. The graphs of the polar curves  $r = 3$  and  $r = 4 - 2\sin \theta$  are shown in the figure above. The curves intersect when  $\theta = \frac{\pi}{6}$  and  $\theta = \frac{5\pi}{6}$ .
- (a) Let  $S$  be the shaded region that is inside the graph of  $r = 3$  and also inside the graph of  $r = 4 - 2\sin \theta$ . Find the area of  $S$ .
- (b) A particle moves along the polar curve  $r = 4 - 2\sin \theta$  so that at time  $t$  seconds,  $\theta = t^2$ . Find the time  $t$  in the interval  $1 \leq t \leq 2$  for which the  $x$ -coordinate of the particle's position is  $-1$ .
- (c) For the particle described in part (b), find the position vector in terms of  $t$ . Find the velocity vector at time  $t = 1.5$ .
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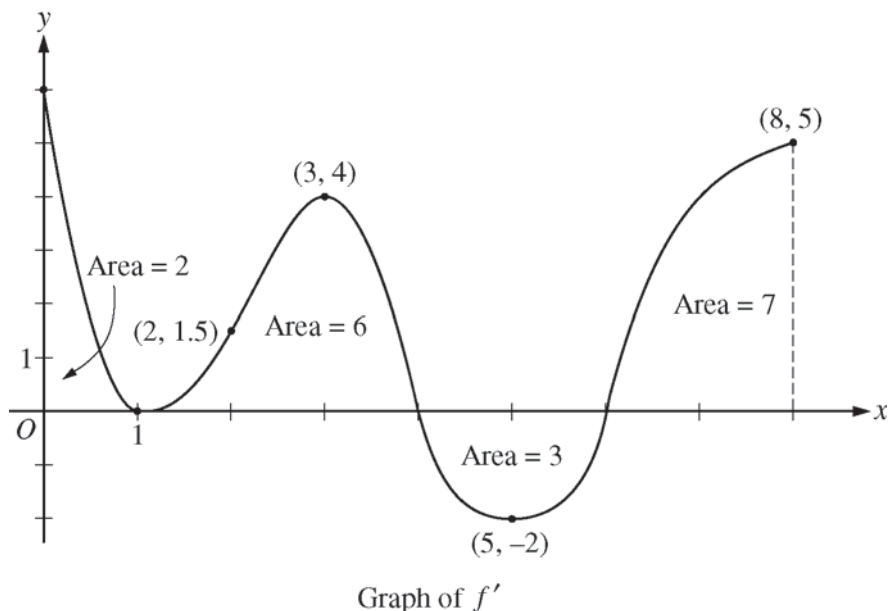
END OF PART A OF SECTION II

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**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

No calculator is allowed for these problems.

$t$ (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by  $B(t) = 16 - 16e^{-0.4t}$ . Using this model, find the rate at which the amount of coffee in the cup is changing when  $t = 5$ .
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4. The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the closed interval  $0 \leq x \leq 8$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$ ,  $x = 3$ , and  $x = 5$ . The areas of the regions between the graph of  $f'$  and the  $x$ -axis are labeled in the figure. The function  $f$  is defined for all real numbers and satisfies  $f(8) = 4$ .
- Find all values of  $x$  on the open interval  $0 < x < 8$  for which the function  $f$  has a local minimum. Justify your answer.
  - Determine the absolute minimum value of  $f$  on the closed interval  $0 \leq x \leq 8$ . Justify your answer.
  - On what open intervals contained in  $0 < x < 8$  is the graph of  $f$  both concave down and increasing? Explain your reasoning.
  - The function  $g$  is defined by  $g(x) = (f(x))^3$ . If  $f(3) = -\frac{5}{2}$ , find the slope of the line tangent to the graph of  $g$  at  $x = 3$ .

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5. Consider the differential equation  $\frac{dy}{dx} = y^2(2x + 2)$ . Let  $y = f(x)$  be the particular solution to the differential equation with initial condition  $f(0) = -1$ .

(a) Find  $\lim_{x \rightarrow 0} \frac{f(x) + 1}{\sin x}$ . Show the work that leads to your answer.

(b) Use Euler's method, starting at  $x = 0$  with two steps of equal size, to approximate  $f\left(\frac{1}{2}\right)$ .

(c) Find  $y = f(x)$ , the particular solution to the differential equation with initial condition  $f(0) = -1$ .

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6. A function  $f$  has derivatives of all orders at  $x = 0$ . Let  $P_n(x)$  denote the  $n$ th-degree Taylor polynomial for  $f$  about  $x = 0$ .
- (a) It is known that  $f(0) = -4$  and that  $P_1\left(\frac{1}{2}\right) = -3$ . Show that  $f'(0) = 2$ .
- (b) It is known that  $f''(0) = -\frac{2}{3}$  and  $f'''(0) = \frac{1}{3}$ . Find  $P_3(x)$ .
- (c) The function  $h$  has first derivative given by  $h'(x) = f(2x)$ . It is known that  $h(0) = 7$ . Find the third-degree Taylor polynomial for  $h$  about  $x = 0$ .
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**STOP**

**END OF EXAM**