



## AP<sup>®</sup> Calculus BC 2012 Free-Response Questions

### About the College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of more than 5,900 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT<sup>®</sup> and the Advanced Placement Program<sup>®</sup>. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

© 2012 The College Board. College Board, Advanced Placement Program, AP, AP Central, SAT, and the acorn logo are registered trademarks of the College Board. Admitted Class Evaluation Service and inspiring minds are trademarks owned by the College Board. All other products and services may be trademarks of their respective owners. Visit the College Board on the Web: [www.collegeboard.org](http://www.collegeboard.org). Permission to use copyrighted College Board materials may be requested online at: [www.collegeboard.org/inquiry/cbpermit.html](http://www.collegeboard.org/inquiry/cbpermit.html).

Visit the College Board on the Web: [www.collegeboard.org](http://www.collegeboard.org).

AP Central is the official online home for the AP Program: [apcentral.collegeboard.org](http://apcentral.collegeboard.org).



2012 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC  
SECTION II, Part A  
Time—30 minutes  
Number of problems—2



A graphing calculator is required for these problems.

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.
- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?
-

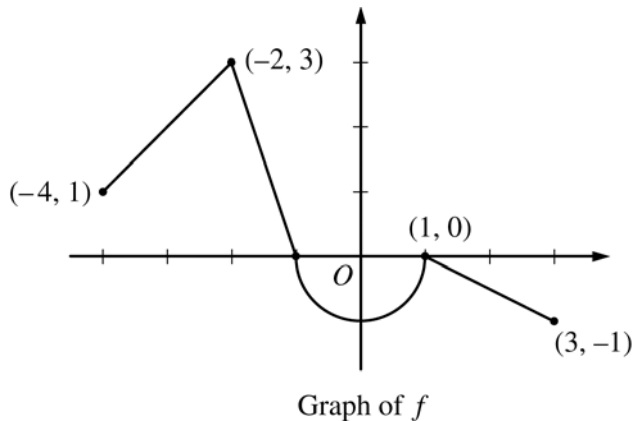
## 2012 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

2. For  $t \geq 0$ , a particle is moving along a curve so that its position at time  $t$  is  $(x(t), y(t))$ . At time  $t = 2$ , the particle is at position  $(1, 5)$ . It is known that  $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$  and  $\frac{dy}{dt} = \sin^2 t$ .
- (a) Is the horizontal movement of the particle to the left or to the right at time  $t = 2$ ? Explain your answer. Find the slope of the path of the particle at time  $t = 2$ .
- (b) Find the  $x$ -coordinate of the particle's position at time  $t = 4$ .
- (c) Find the speed of the particle at time  $t = 4$ . Find the acceleration vector of the particle at time  $t = 4$ .
- (d) Find the distance traveled by the particle from time  $t = 2$  to  $t = 4$ .
- 

**END OF PART A OF SECTION II**

**CALCULUS BC**  
**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

No calculator is allowed for these problems.



3. Let  $f$  be the continuous function defined on  $[-4, 3]$  whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let  $g$  be the function given by  $g(x) = \int_1^x f(t) dt$ .
- Find the values of  $g(2)$  and  $g(-2)$ .
  - For each of  $g'(-3)$  and  $g''(-3)$ , find the value or state that it does not exist.
  - Find the  $x$ -coordinate of each point at which the graph of  $g$  has a horizontal tangent line. For each of these points, determine whether  $g$  has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
  - For  $-4 < x < 3$ , find all values of  $x$  for which the graph of  $g$  has a point of inflection. Explain your reasoning.

2012 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

$x$	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function  $f$  is twice differentiable for  $x > 0$  with  $f(1) = 15$  and  $f''(1) = 20$ . Values of  $f'$ , the derivative of  $f$ , are given for selected values of  $x$  in the table above.
- (a) Write an equation for the line tangent to the graph of  $f$  at  $x = 1$ . Use this line to approximate  $f(1.4)$ .
- (b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate  $\int_1^{1.4} f'(x) dx$ . Use the approximation for  $\int_1^{1.4} f'(x) dx$  to estimate the value of  $f(1.4)$ . Show the computations that lead to your answer.
- (c) Use Euler's method, starting at  $x = 1$  with two steps of equal size, to approximate  $f(1.4)$ . Show the computations that lead to your answer.
- (d) Write the second-degree Taylor polynomial for  $f$  about  $x = 1$ . Use the Taylor polynomial to approximate  $f(1.4)$ .
-

2012 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

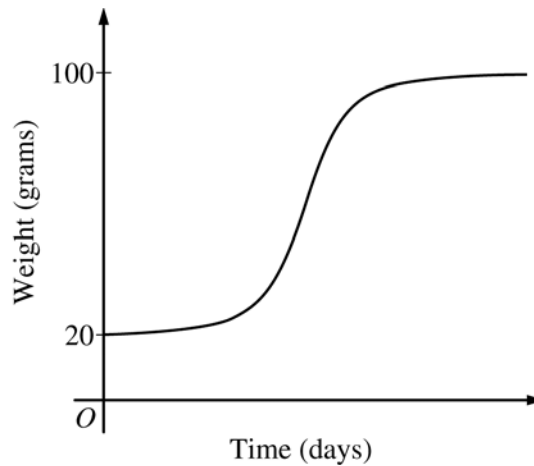
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time  $t = 0$ , when the bird is first weighed, its weight is 20 grams. If  $B(t)$  is the weight of the bird, in grams, at time  $t$  days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let  $y = B(t)$  be the solution to the differential equation above with initial condition  $B(0) = 20$ .

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

- (b) Find  $\frac{d^2B}{dt^2}$  in terms of  $B$ . Use  $\frac{d^2B}{dt^2}$  to explain why the graph of  $B$  cannot resemble the following graph.



- (c) Use separation of variables to find  $y = B(t)$ , the particular solution to the differential equation with initial condition  $B(0) = 20$ .
-

2012 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS



6. The function  $g$  has derivatives of all orders, and the Maclaurin series for  $g$  is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \dots$$

- (a) Using the ratio test, determine the interval of convergence of the Maclaurin series for  $g$ .
  - (b) The Maclaurin series for  $g$  evaluated at  $x = \frac{1}{2}$  is an alternating series whose terms decrease in absolute value to 0. The approximation for  $g\left(\frac{1}{2}\right)$  using the first two nonzero terms of this series is  $\frac{17}{120}$ . Show that this approximation differs from  $g\left(\frac{1}{2}\right)$  by less than  $\frac{1}{200}$ .
  - (c) Write the first three nonzero terms and the general term of the Maclaurin series for  $g'(x)$ .
- 

**STOP**

**END OF EXAM**