Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

16 pages

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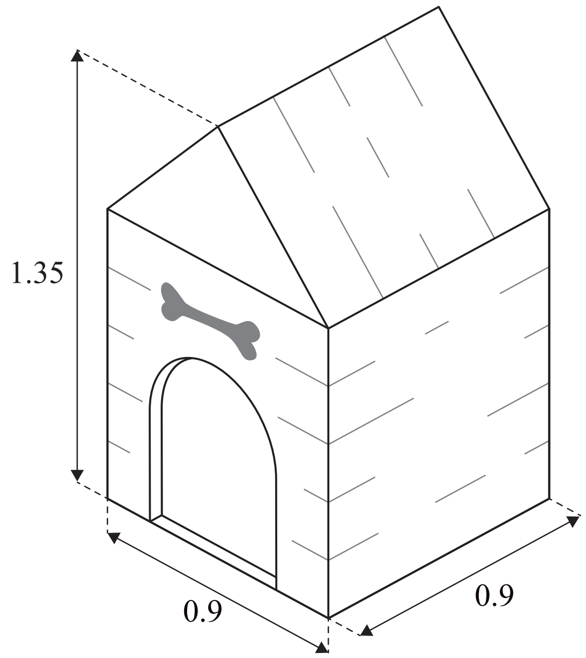
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

The front view of a doghouse is made up of a square with an isosceles triangle on top. The doghouse is 1.35 m high and 0.9 m wide, and sits on a square base.

diagram not to scale



The top of the rectangular surfaces of the roof of the doghouse are to be painted.

Find the area to be painted.

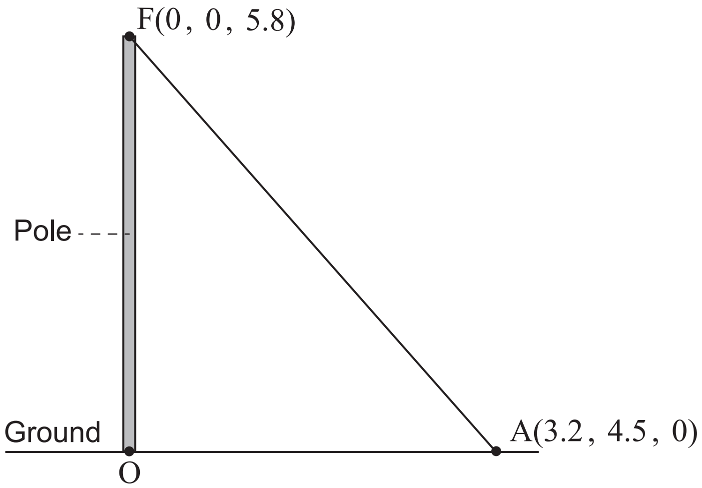
(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 4]

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0, 0, 5.8)$. All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F.

One of the ropes is attached to the ground at a point A with coordinates $(3.2, 4.5, 0)$. The rope forms a straight line from A to F.

(a) Find the length of the rope connecting A to F. [2]

(b) Find $\mathrm{F\hat{A}O}$, the angle the rope makes with the ground. [2]

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3. [Maximum mark: 5]

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

(b) Find the value of t when the ball hits the ground. [2]

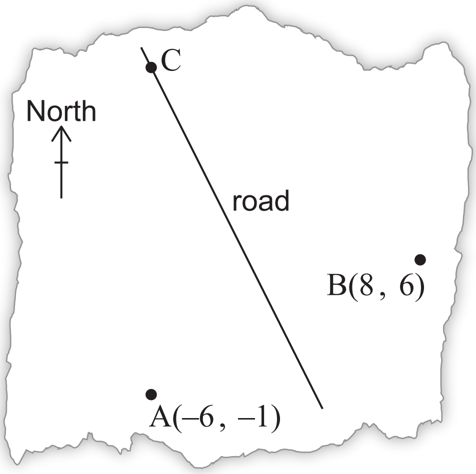
(c) State an appropriate domain for t in this model. [2]

4. [Maximum mark: 7]

Three towns, A, B and C are represented as coordinates on a map, where the x and y axes represent the distances east and north of an origin, respectively, measured in kilometres.

Town A is located at $(-6, -1)$ and town B is located at $(8, 6)$. A road runs along the perpendicular bisector of $[AB]$. This information is shown in the following diagram.

diagram not to scale



(a) Find the equation of the line that the road follows. [5]

Town C is due north of town A and the road passes through town C.

(b) Find the y-coordinate of town C. [2]

5. [Maximum mark: 5]

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, \$)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was $\$7816$.
- There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x, the number of child tickets sold be y, and the number of student tickets sold be z.

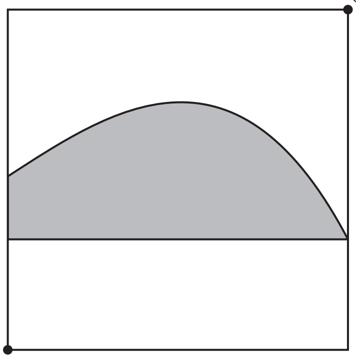
(a) Write down three equations that express the information given above. [3]

(b) Find the number of each type of ticket sold. [2]

6. [Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1\,,\,-1)$ and the top right corner has coordinates $(2\,,\,2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x-axis. Distances are measured in metres.

(a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region. [3]

x	-1	0	1	2
y	0.6	1.2	1.2	0

The artist used the equation $y = \frac{-x^3 - 3x^2 + 4x + 12}{10}$ to draw the curve.

(b) Find the exact area of the shaded region in the painting. [2]

(c) Find the area of the unshaded region in the painting. [2]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 7]

Leo is investigating whether a six-sided die is fair. He rolls the die 60 times and records the observed frequencies in the following table:

Number on die	1	2	3	4	5	6
Observed frequency	8	7	6	15	12	12

Leo carries out a χ^2 goodness of fit test at a 5% significance level.

(a) Write down the null and alternative hypotheses. [1]

(b) Write down the degrees of freedom. [1]

(c) Write down the expected frequency of rolling a 1. [1]

(d) Find the p-value for the test. [2]

(e) State the conclusion of the test. Give a reason for your answer. [2]

8. [Maximum mark: 6]

A factory produces bags of sugar with a labelled weight of 500 g. The weights of the bags are normally distributed with a mean of 500 g and a standard deviation of 3 g.

(a) Write down the percentage of bags that weigh more than 500 g. [1]

A bag that weighs less than 495 g is rejected by the factory for being underweight.

(b) Find the probability that a randomly chosen bag is rejected for being underweight. [2]

A bag that weighs more than k grams is rejected by the factory for being overweight.
The factory rejects 2% of bags for being overweight.

(c) Find the value of k. [3]

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9. [Maximum mark: 7]

The function f is defined by $f(x) = \frac{2}{x} + 3x^2 - 3$, $x \neq 0$.

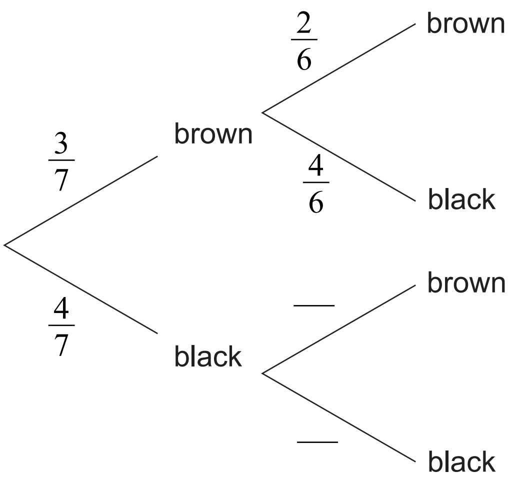
(a) Find $f'(x)$. [3]

(b) Find the equation of the normal to the curve $y = f(x)$ at $(1, 2)$ in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [4]

10. [Maximum mark: 6]

Karl has three brown socks and four black socks in his drawer. He takes two socks at random from the drawer.

(a) Complete the tree diagram. [1]



(b) Find the probability that Karl takes two socks of the same colour. [2]

(c) Given that Karl has two socks of the same colour find the probability that he has two brown socks. [3]

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11. [Maximum mark: 8]

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$$\log_{10} N = a - M, \text{ for some } a \in \mathbb{R}.$$

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

(c) Given $0 < M < 8$, find the range for N. [2]

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(d) Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year. [2]

12. [Maximum mark: 6]

A company's profit per year was found to be changing at a rate of

$$\frac{dP}{dt} = 3t^2 - 8t$$

where P is the company's profit in thousands of dollars and t is the time since the company was founded, measured in years.

(a) Determine whether the profit is increasing or decreasing when $t = 2$. [2]

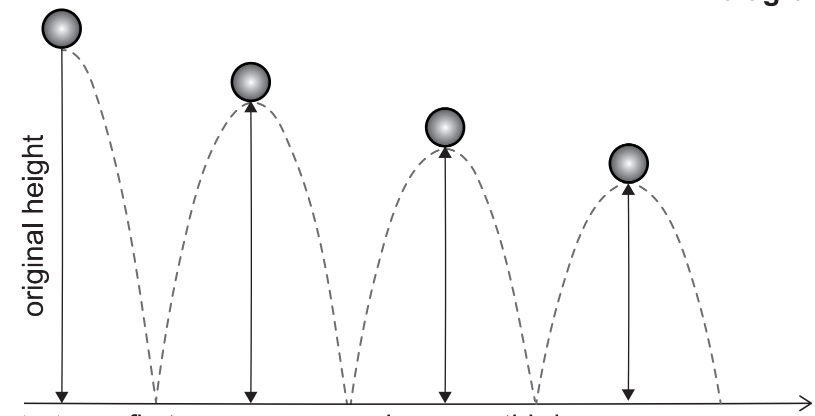
One year after the company was founded, the profit was 4 thousand dollars.

(b) Find an expression for $P(t)$, when $t \geq 0$. [4]

13. [Maximum mark: 7]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.

diagram not to scale



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

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