Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 4]

Katya approximates π, correct to four decimal places, by using the following expression.

$$3 + \frac{1}{6 + \frac{13}{16}}$$

(a) Calculate Katya's approximation of π, correct to four decimal places. [2]

(b) Calculate the percentage error in using Katya's four decimal place approximation of π, compared to the exact value of π in your calculator. [2]

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2. [Maximum mark: 4]

Deb used a thermometer to record the maximum daily temperature over ten consecutive days. Her results, in degrees Celsius ($^\circ C$), are shown below.

$$14, 15, 14, 11, 10, 9, 14, 15, 16, 13$$

For this data set, find the value of

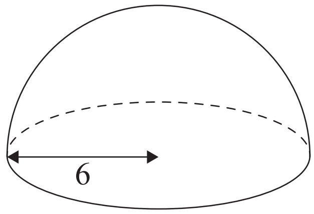
(a) the mode. [1]

(b) the mean. [2]

(c) the standard deviation. [1]

3. [Maximum mark: 6]

A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is 6 mm.



(a) Calculate the **total** surface area of one piece of candy. [4]

The total surface of the candy is coated in chocolate. It is known that 1 gram of the chocolate covers an area of $240\,\text{mm}^2$.

(b) Calculate the weight of chocolate required to coat one piece of candy. [2]

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4. [Maximum mark: 7]

The price of gas at Leon's gas station is $\$1.50$ per litre. If a customer buys a minimum of 10 litres, a discount of $\$5$ is applied.

This can be modelled by the following function, L, which gives the total cost when buying a minimum of 10 litres at Leon's gas station.

$$L(x) = 1.50x - 5\,,\ x \geq 10$$

where x is the number of litres of gas that a customer buys.

(a) Find the total cost of buying 40 litres of gas at Leon's gas station. [2]

(b) Find $L^{-1}(70)$. [2]

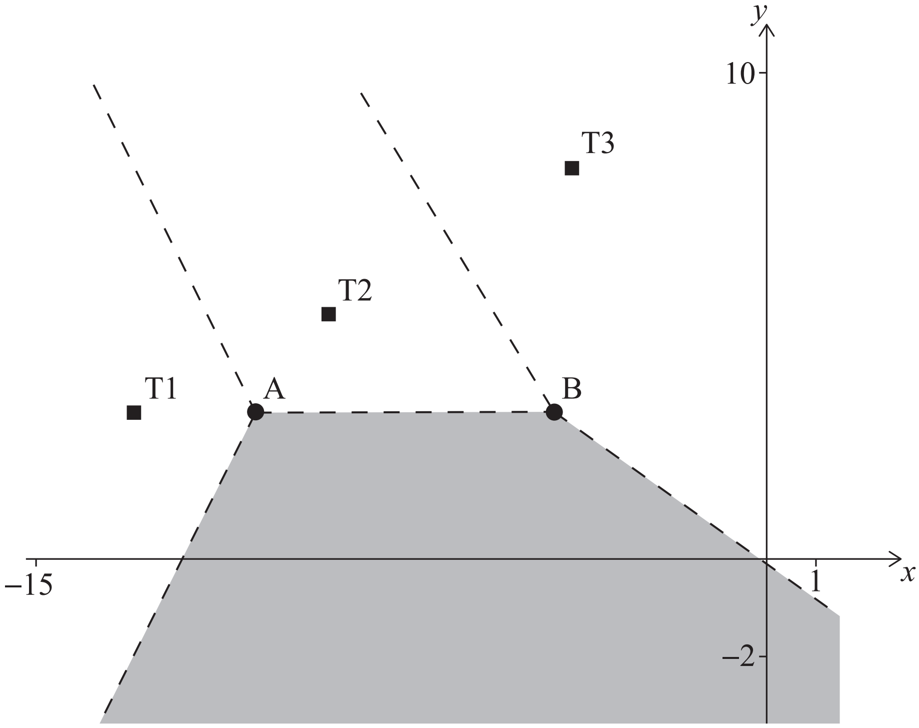
The price of gas at Erica's gas station is $\$1.30$ per litre. A customer must buy a minimum of 10 litres of gas. The total cost at Erica's gas station is cheaper than Leon's gas station when $x > k$.

(c) Find the minimum value of k. [3]

5. [Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.
Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 5]

Arriane has geese on her farm. She claims the mean weight of eggs from her black geese is less than the mean weight of eggs from her white geese.

She recorded the weights of eggs, in grams, from a random selection of geese. The data is shown in the table.

Weights of eggs from black geese	136	134	142	141	128	126
Weights of eggs from white geese	135	138	141	140	136	134

In order to test her claim, Arriane performs a t-test at a 10% level of significance. It is assumed that the weights of eggs are normally distributed and the samples have equal variances.

(a) State, in words, the null hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) State whether the result of the test supports Arriane's claim. Justify your reasoning. [2]

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7. [Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\,e^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]

8. [Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\,\text{m}$. On each subsequent day, Charlie ran $100\,\text{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme. [5]

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

(b) Find the value of n. [3]

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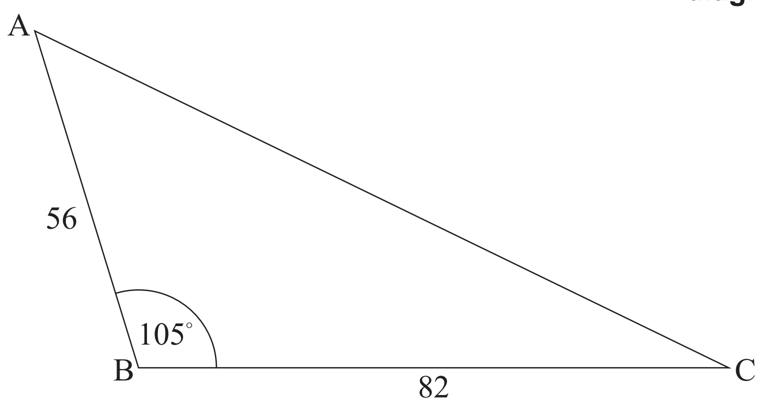
9. [Maximum mark: 5]

A triangular field ABC is such that $AB = 56\,m$ and $BC = 82\,m$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

diagram not to scale



Calculate the maximum possible area of the field.

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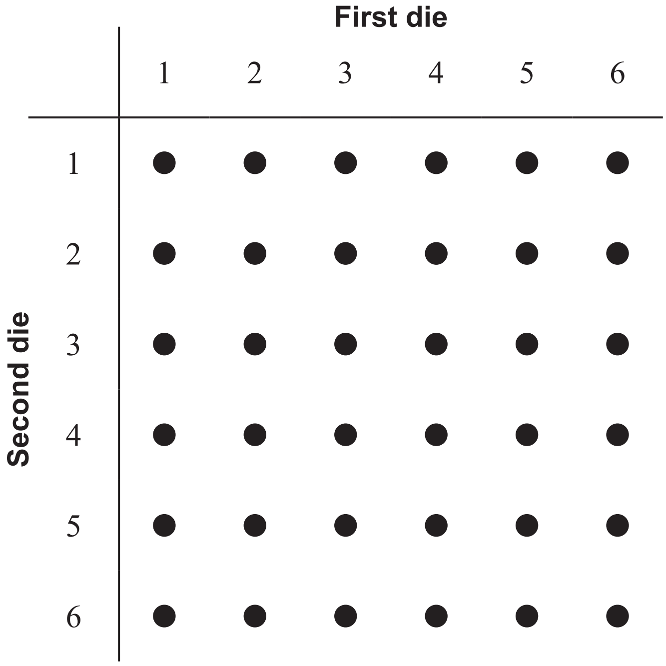
10. [Maximum mark: 7]

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable "the score in a game".

(a) Complete the table to show the probability distribution of T. [2]

t	1	2	3	4	5	6
$\mathrm{P}(T=t)$						

(b) Find the probability that

(i) a player scores at least 3 in a game.

(ii) a player scores 6, given that they scored at least 3. [3]

(c) Find the expected score of a game. [2]

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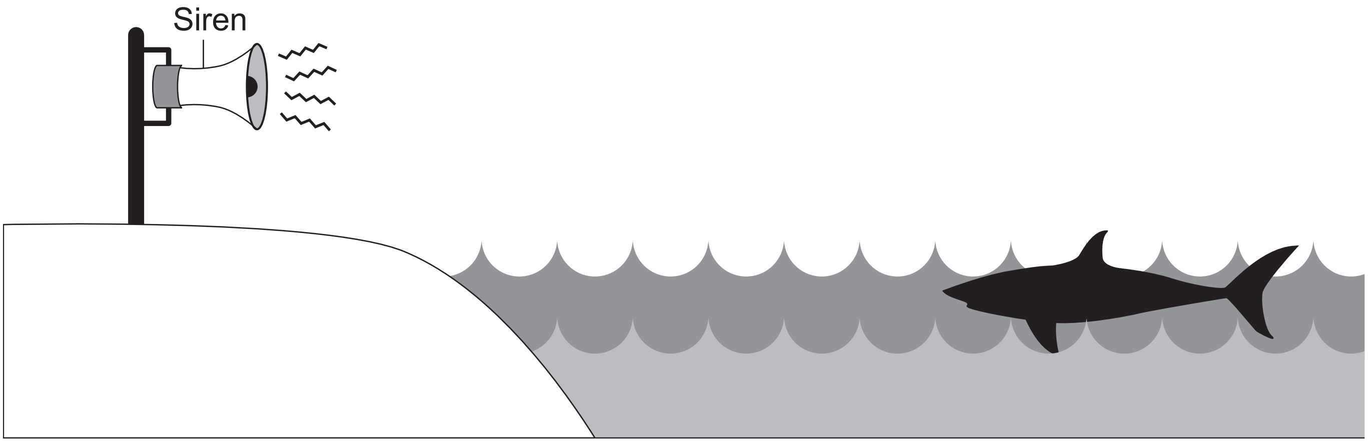
(Question 10 continued)

11. [Maximum mark: 6]

If a shark is spotted near to Brighton beach, a lifeguard will activate a siren to warn swimmers.

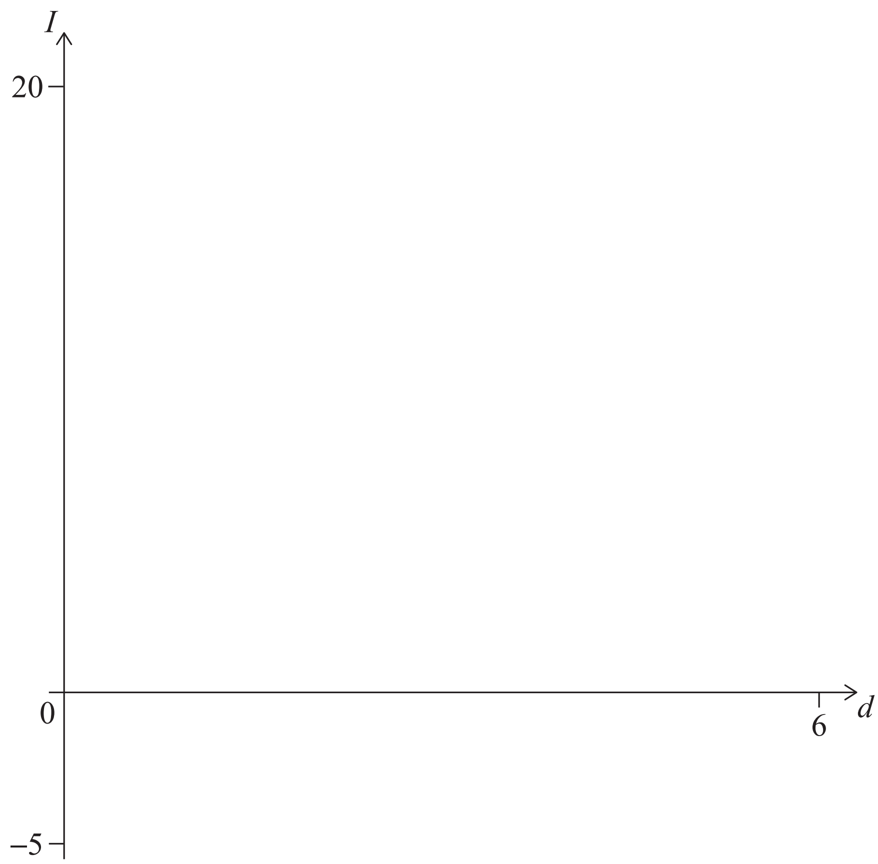


The sound intensity, I, of the siren varies inversely with the square of the distance, d, from the siren, where $d > 0$.

It is known that at a distance of 1.5 metres from the siren, the sound intensity is 4 watts per square metre $(\mathrm{W\,m^{-2}})$.

(a) Show that $I = \dfrac{9}{d^2}$. [2]

(b) Sketch the curve of I on the axes below showing clearly the point $(1.5, 4)$. [2]



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(Question 11 continued)

Whilst swimming, Scarlett can hear the siren only if the sound intensity at her location is greater than $1.5 \times 10^{-6}\,\mathrm{W\,m^{-2}}$.

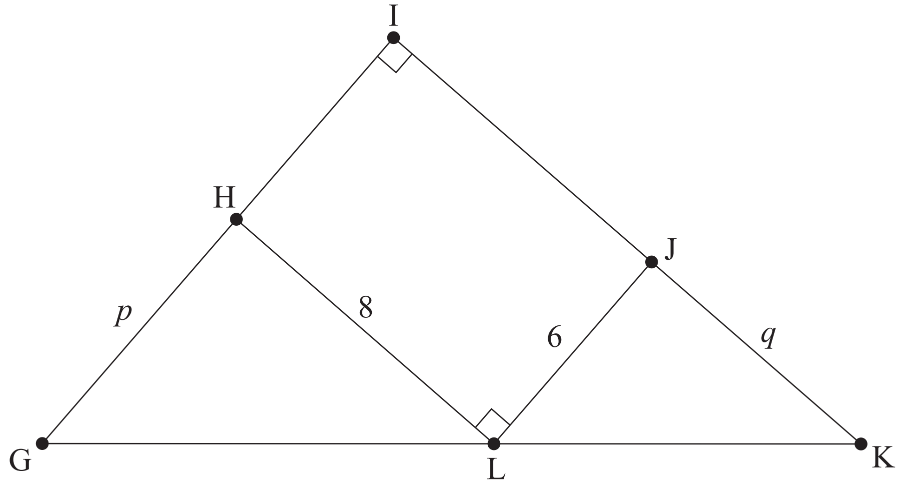
(c) Find the values of d where Scarlett cannot hear the siren. [2]

12. [Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is $A\,\text{cm}^2$.

(a) (i) Find A in terms of p and q.

(ii) Show that $A = \frac{192}{q} + 3q + 48$. [4]

(b) Find $\frac{dA}{dq}$. [2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q.

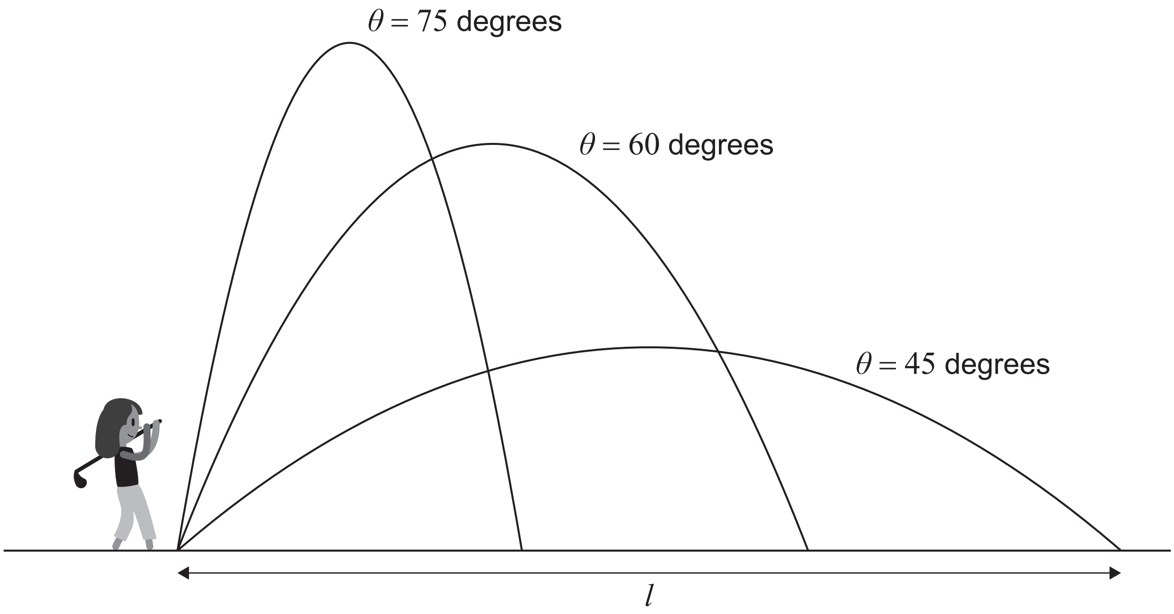
(ii) Hence, or otherwise, find this value of q. [2]

(This question continues on the following page)

(Question 12 continued)

13. [Maximum mark: 8]

Sieun hits golf balls into the air. Each time she hits a ball she records θ, the angle at which the ball is launched into the air, and l, the horizontal distance, in metres, which the ball travels from the point of contact to the first time it lands. The diagram below represents this information.



Sieun analyses her results and concludes:

$$\frac{\mathrm{d}l}{\mathrm{d}\theta} = -0.2\theta + 9,\ 35^\circ \le \theta \le 75^\circ.$$

(a) Determine whether the graph of l against θ is increasing or decreasing at $\theta = 50^\circ$. [3]

Sieun observes that when the angle is 40°, the ball will travel a horizontal distance of $205.5\,\text{m}$.

(b) Find an expression for the function $l(\theta)$. [5]

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(Question 13 continued)

References: