

Mathematics: applications and interpretation
Higher level
Paper 2

Monday 9 May 2022 (morning)

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Blank page

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

The scores of the eight highest scoring countries in the 2019 Eurovision song contest are shown in the following table.

	Eurovision score
Netherlands	498
Italy	472
Russia	370
Switzerland	364
Sweden	334
Norway	331
North Macedonia	305
Azerbaijan	302

- (a) For this data, find
 - (i) the upper quartile.
 - (ii) the interquartile range. [4]
- (b) Determine if the Netherlands' score is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 1 continued)

Chester is investigating the relationship between the highest-scoring countries' Eurovision score and their population size to determine whether population size can reasonably be used to predict a country's score.

The populations of the countries, to the nearest million, are shown in the table.

	Population (x) (millions)	Eurovision score (y)
Netherlands	17	498
Italy	60	472
Russia	145	370
Switzerland	9	364
Sweden	10	334
Norway	5	331
North Macedonia	2	305
Azerbaijan	10	302

Chester finds that, for this data, the Pearson's product moment correlation coefficient is $r = 0.249$.

- (c) State whether it would be appropriate for Chester to use the equation of a regression line for y on x to predict a country's Eurovision score. Justify your answer.

[2]

Chester then decides to find the Spearman's rank correlation coefficient for this data, and creates a table of ranks.

	Population rank (to the nearest million)	Eurovision score rank
Netherlands	3	1
Italy	2	2
Russia	1	3
Switzerland	a	4
Sweden	b	5
Norway	7	6
North Macedonia	8	7
Azerbaijan	c	8

(This question continues on the following page)

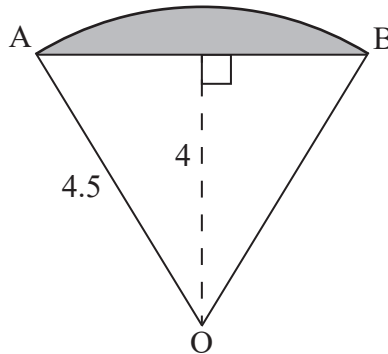
(Question 1 continued)

- (d) Write down the value of:
- (i) a ,
 - (ii) b ,
 - (iii) c . [3]
- (e) (i) Find the value of the Spearman's rank correlation coefficient r_s .
(ii) Interpret the value obtained for r_s . [3]
- (f) When calculating the ranks, Chester incorrectly read the Netherlands' score as 478. Explain why the value of the Spearman's rank correlation r_s does not change despite this error. [1]

2. [Maximum mark: 17]

A sector of a circle, centre O and radius 4.5 m, is shown in the following diagram.

diagram not to scale



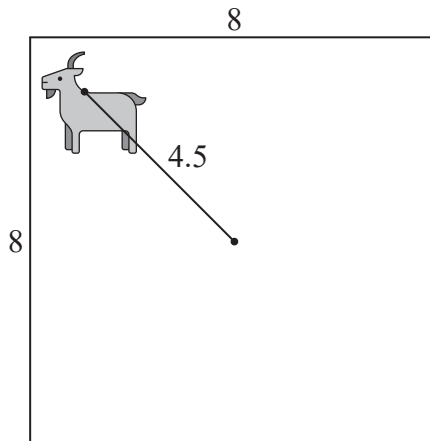
(a) (i) Find the angle \hat{AOB} .

(ii) Find the area of the shaded segment.

[8]

A square field with side 8 m has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to 4.5 m from the post.

diagram not to scale



(b) Find the area of the field that can be reached by the goat.

[4]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

The goat eats grass at the rate of $\frac{dV}{dt} = 0.3te^{-t}$.

(c) Find the value of t at which the goat is eating grass at the greatest rate.

[2]

The goat is tied in the field for 8 hours.

(d) Find the total volume of grass eaten by the goat during this time.

[3]

3. [Maximum mark: 13]

A Principal would like to compare the students in his school with a national standard. He decides to give a test to eight students made up of four boys and four girls. One of the teachers offers to find the volunteers from his class.

- (a) Name the type of sampling that best describes the method used by the Principal. [1]

The marks out of 40, for the students who took the test, are:

25, 29, 38, 37, 12, 18, 27, 31.

- (b) For the eight students find

(i) the mean mark.

(ii) the standard deviation of the marks. [3]

The national standard mark is 25.2 out of 40.

- (c) Perform an appropriate test at the 5% significance level to see if the mean marks achieved by the students in the school are higher than the national standard. It can be assumed that the marks come from a normal population. [5]

- (d) State one reason why the test might not be valid. [1]

Two additional students take the test at a later date and the mean mark for all ten students is 28.1 and the standard deviation is 8.4.

For further analysis, a standardized score out of 100 for the ten students is obtained by multiplying the scores by 2 and adding 20.

- (e) For the ten students, find

(i) their mean standardized score.

(ii) the standard deviation of their standardized score. [3]

4. [Maximum mark: 13]

A particle moves such that its displacement, x metres, from a point O at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

- (a) (i) Use the substitution $y = \frac{dx}{dt}$ to show that this equation can be written as

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

- (ii) Find the eigenvalues for the matrix $\begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix}$.

- (iii) Hence state the long-term velocity of the particle.

[5]

The equation for the motion of the particle is amended to

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 3t + 4.$$

- (b) (i) Use the substitution $y = \frac{dx}{dt}$ to write the differential equation as a system of coupled, first order differential equations.

When $t = 0$ the particle is stationary at O.

- (ii) Use Euler's method with a step length of 0.1 to find the displacement of the particle when $t = 1$.

- (iii) Find the long-term velocity of the particle.

[8]

5. [Maximum mark: 15]

The aircraft for a particular flight has 72 seats. The airline’s records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

- (a) The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. [3]
- (b) (i) Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. [2]
- (ii) Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. [2]

Each passenger pays \$150 for a ticket. If too many passengers arrive, then the airline will give \$300 in compensation to each passenger that cannot board.

- (c) Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. [8]

6. [Maximum mark: 18]

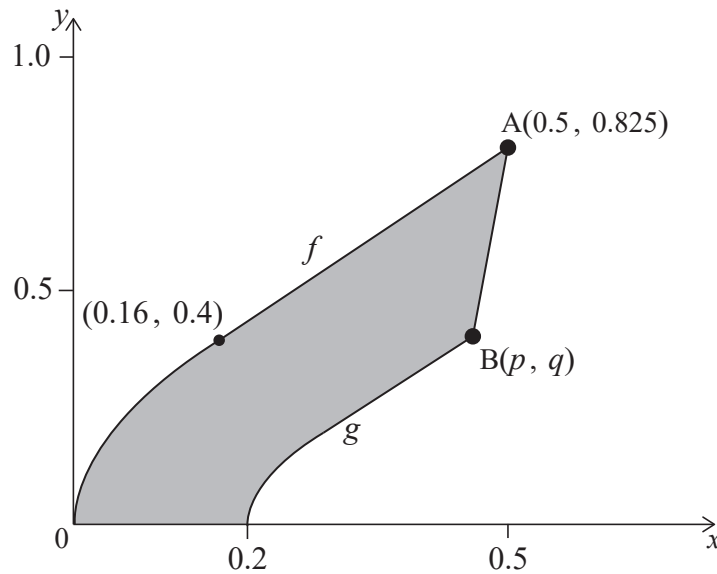
Consider the curve $y = \sqrt{x}$.

(a) (i) Find $\frac{dy}{dx}$.

(ii) Hence show that the equation of the tangent to the curve at the point $(0.16, 0.4)$ is $y = 1.25x + 0.2$.

[4]

The shape of a piece of metal can be modelled by the region bounded by the functions f , g , the x -axis and the line segment $[AB]$, as shown in the following diagram. The units on the x and y axes are measured in metres.



The piecewise function f is defined by

$$f(x) = \begin{cases} \sqrt{x} & 0 \leq x \leq 0.16 \\ 1.25x + 0.2 & 0.16 < x \leq 0.5 \end{cases}$$

The graph of g is obtained from the graph of f by:

- a stretch scale factor of $\frac{1}{2}$ in the x direction,
- followed by a stretch scale factor $\frac{1}{2}$ in the y direction,
- followed by a translation of 0.2 units to the right.

Point A lies on the graph of f and has coordinates $(0.5, 0.825)$. Point B is the image of A under the given transformations and has coordinates (p, q) .

(b) Find the value of p and the value of q .

[2]

(This question continues on the following page)

(Question 6 continued)

The piecewise function g is given by

$$g(x) = \begin{cases} h(x) & 0.2 \leq x \leq a \\ 1.25x + b & a < x \leq p \end{cases}$$

(c) Find

(i) an expression for $h(x)$.

(ii) the value of a .

(iii) the value of b .

[5]

(d) (i) Find the area enclosed by $y = f(x)$, the x -axis and the line $x = 0.5$.

The area enclosed by $y = g(x)$, the x -axis and the line $x = p$ is 0.0627292 m^2 correct to six significant figures.

(ii) Find the area of the shaded region on the diagram.

[7]

7. [Maximum mark: 18]

A transformation, T , of a plane is represented by $\mathbf{r}' = \mathbf{P}\mathbf{r} + \mathbf{q}$, where \mathbf{P} is a 2×2 matrix, \mathbf{q} is a 2×1 vector, \mathbf{r} is the position vector of a point in the plane and \mathbf{r}' the position vector of its image under T .

The triangle OAB has coordinates $(0, 0)$, $(0, 1)$ and $(1, 0)$. Under T , these points are transformed to $(0, 1)$, $(\frac{1}{4}, 1 + \frac{\sqrt{3}}{4})$ and $(\frac{\sqrt{3}}{4}, \frac{3}{4})$ respectively.

- (a) (i) By considering the image of $(0, 0)$, find \mathbf{q} .
- (ii) By considering the image of $(1, 0)$ and $(0, 1)$, show that [6]

$$\mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix}.$$

\mathbf{P} can be written as $\mathbf{P} = \mathbf{R}\mathbf{S}$, where \mathbf{S} and \mathbf{R} are matrices.

\mathbf{S} represents an enlargement with scale factor 0.5, centre $(0, 0)$.

\mathbf{R} represents a rotation about $(0, 0)$.

- (b) Write down the matrix \mathbf{S} . [1]
- (c) (i) Use $\mathbf{P} = \mathbf{R}\mathbf{S}$ to find the matrix \mathbf{R} .
- (ii) Hence find the angle and direction of the rotation represented by \mathbf{R} . [7]

The transformation T can also be described by an enlargement scale factor $\frac{1}{2}$, centre (a, b) , followed by a rotation about the same centre (a, b) .

- (d) (i) Write down an equation satisfied by $\begin{pmatrix} a \\ b \end{pmatrix}$.
- (ii) Find the value of a and the value of b . [4]

References:

2. mynamepong, n.d. Goat [image online] Available at: <https://thenounproject.com/term/goat/1761571/>
 This file is licensed under the Creative Commons Attribution-ShareAlike 3.0 Unported (CC BY-SA 3.0) <https://creativecommons.org/licenses/by-sa/3.0/deed.en> [Accessed 22 April 2010] Source adapted.