Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics: applications and interpretation
# Higher level
# Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

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2 hours

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

20 pages

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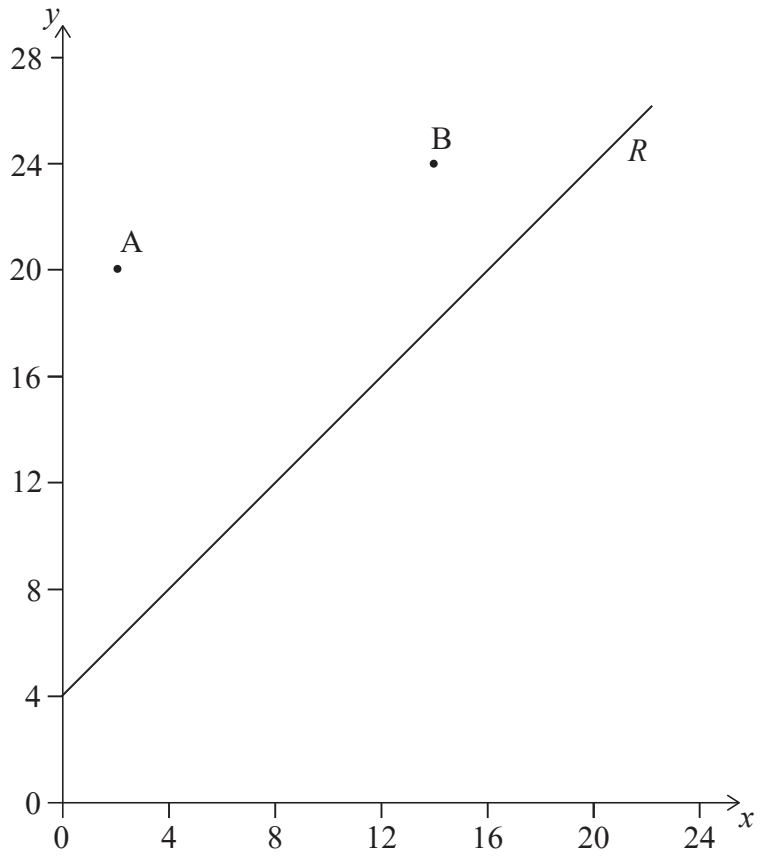
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 7]

Two schools are represented by points $\mathrm{A}(2,\ 20)$ and $\mathrm{B}(14,\ 24)$ on the graph below. A road, represented by the line $R$ with equation $-x+y=4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.




(a) Find the equation of the perpendicular bisector of $[\mathrm{AB}]$. Give your equation in the form $y=mx+c$. [5]

(b) Determine the coordinates of the point on $R$ where the bus stop should be located. [2]

**(This question continues on the following page)**

**(Question 1 continued)**

**2.** [Maximum mark: 7]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \le x \le 7$, $x \ne -5$.

(a) Find the range of $f$. [3]

(b) Find an expression for the inverse function $f^{-1}(x)$. The domain is not required. [3]

(c) Write down the range of $f^{-1}(x)$. [1]

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**3.** [Maximum mark: 6]

At Springfield University, the weights, in kg, of 10 chinchilla rabbits and 10 sable rabbits were recorded. The aim was to find out whether chinchilla rabbits are generally heavier than sable rabbits. The results obtained are summarized in the following table.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Weight of chinchilla rabbits, kg** | 4.9 | 4.2 | 4.1 | 4.4 | 4.3 | 4.6 | 4.0 | 4.7 | 4.5 | 4.4 |
| **Weight of sable rabbits, kg** | 4.2 | 4.1 | 4.1 | 4.2 | 4.5 | 4.4 | 4.5 | 3.9 | 4.2 | 4.0 |

A $t$-test is to be performed at the 5 % significance level.

(a) Write down the null and alternative hypotheses. [2]

(b) Find the $p$-value for this test. [2]

(c) Write down the conclusion to the test. Give a reason for your answer. [2]

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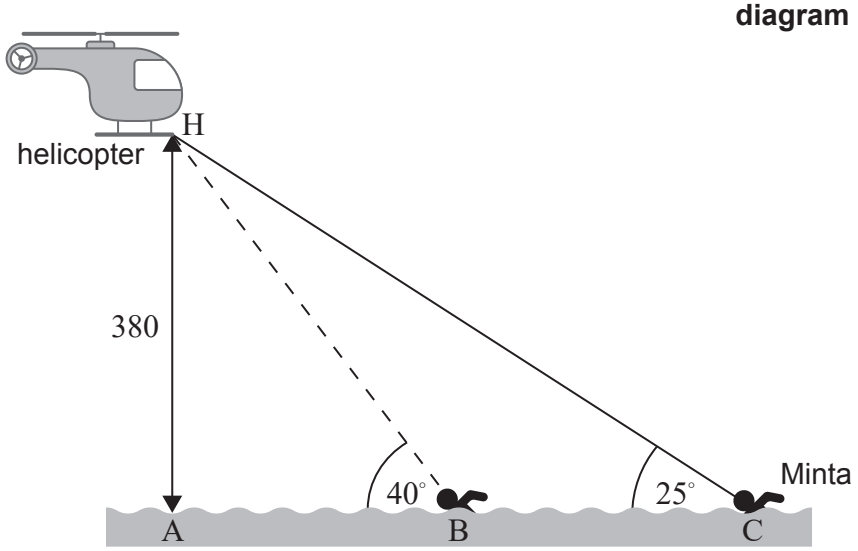
**4.** [Maximum mark: 6]

The diagram below shows a helicopter hovering at point $\mathrm{H}$, $380\,\mathrm{m}$ vertically above a lake. Point $\mathrm{A}$ is the point on the surface of the lake, directly below the helicopter.

**diagram not to scale**




Minta is swimming at a constant speed in the direction of point $\mathrm{A}$. Minta observes the helicopter from point $\mathrm{C}$ as she looks upward at an angle of $25^\circ$. After $15$ minutes, Minta is at point $\mathrm{B}$ and she observes the same helicopter at an angle of $40^\circ$.

(a) Find the distance from $\mathrm{A}$ to $\mathrm{C}$. [2]

(b) Find the distance from $\mathrm{B}$ to $\mathrm{C}$. [3]

(c) Find Minta's speed, in metres per hour. [1]

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**5.** [Maximum mark: 5]

Roger buys a new laptop for himself at a cost of £495. At the same time, he buys his daughter Chloe a higher specification laptop at a cost of £2200.

It is anticipated that Roger's laptop will depreciate at a rate of $10\%$ per year, whereas Chloe's laptop will depreciate at a rate of $15\%$ per year.

(a) Estimate the value of Roger's laptop after $5$ years. [2]

Roger and Chloe's laptops will have the same value $k$ years after they were purchased.

(b) Find the value of $k$. [2]

(c) Comment on the validity of your answer to part (b). [1]

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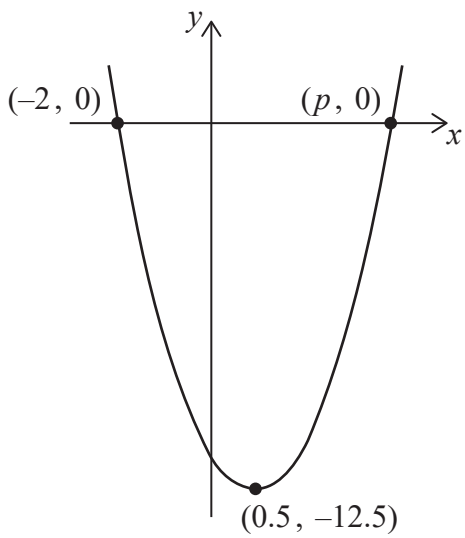
**6.** [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the $x$-axis at two points, $(-2, 0)$ and $(p, 0)$.

**diagram not to scale**




(a) Find the value of $p$. [1]

(b) Find the value of

(i) $a$.

(ii) $b$.

(iii) $c$. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

**(This question continues on the following page)**

**(Question 6 continued)**

**7.** [Maximum mark: 6]

A meteorologist models the height of a hot air balloon launched from the ground. The model assumes the balloon travels vertically upwards and travels $450$ metres in the first minute.

Due to the decrease in temperature as the balloon rises, the balloon will continually slow down. The model suggests that each minute the balloon will travel only $82\%$ of the distance travelled in the previous minute.

(a) Find how high the balloon will travel in the first $10$ minutes after it is launched. [3]

(b) The balloon is required to reach a height of at least $2520$ metres.

Determine whether it will reach this height. [2]

(c) Suggest a limitation of the given model. [1]

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**8.** [Maximum mark: 7]

Two lines $L_1$ and $L_2$ are given by the following equations, where $p \in \mathbb{R}$.

$$L_1 : r = \begin{pmatrix} 2 \\ p+9 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ p+12 \end{pmatrix} + \mu \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix}$$

It is known that $L_1$ and $L_2$ are perpendicular.

(a) Find the possible value(s) for $p$. [3]

(b) In the case that $p < 0$, determine whether the lines intersect. [4]

**9.** [Maximum mark: 8]

A newspaper vendor in Singapore is trying to predict how many copies of *The Straits Times* they will sell. The vendor forms a model to predict the number of copies sold each weekday. According to this model, they expect the same number of copies will be sold each day.

To test the model, they record the number of copies sold each weekday during a particular week. This data is shown in the table.

| **Day** | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| **Number of copies sold** | 74 | 97 | 91 | 86 | 112 |

A goodness of fit test at the $5\%$ significance level is used on this data to determine whether the vendor's model is suitable. The critical value for the test is $9.49$.

(a) Find an estimate for how many copies the vendor expects to sell each day. [1]

(b) (i) State the null and alternative hypotheses for this test.

(ii) Write down the degrees of freedom for this test.

(iii) Write down the conclusion to the test. Give a reason for your answer. [7]

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**10.** [Maximum mark: 6]

A manufacturer of chocolates produces them in individual packets, claiming to have an average of $85$ chocolates per packet.

Talha bought $30$ of these packets in order to check the manufacturer's claim.

Given that the number of individual chocolates is $x$, Talha found that, from his $30$ packets, $\sum x = 2506$ and $\sum x^2 = 209\,738$.

(a) Find an unbiased estimate for the mean number $(\mu)$ of chocolates per packet. [1]

(b) Use the formula $s_{n-1}^2 = \dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$ to determine an unbiased estimate for the variance of the number of chocolates per packet. [2]

(c) Find a $95\,\%$ confidence interval for $\mu$. You may assume that all conditions for a confidence interval have been met. [2]

(d) Suggest, with justification, a valid conclusion that Talha could make. [1]

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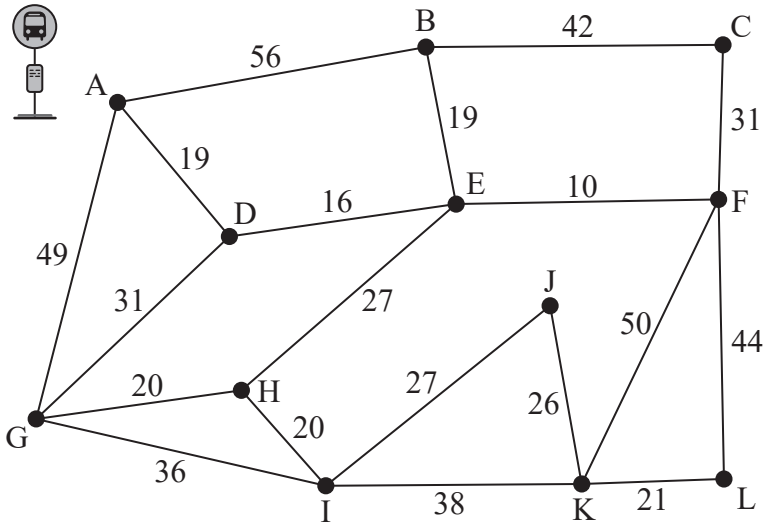
**11.** [Maximum mark: 7]

The diagram below shows a network of roads in a small village with the weights indicating the distance of each road, in metres, and junctions indicated with letters.




Musab is required to deliver leaflets to every house on each road. He wishes to minimize his total distance.

(a) Musab starts and finishes from the village bus-stop at A. Determine the total distance Musab will need to walk. [5]

Instead of having to catch the bus to the village, Musab's sister offers to drop him off at any junction and pick him up at any other junction of his choice.

(b) Explain which junctions Musab should choose as his starting and finishing points. [2]

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**12.** [Maximum mark: 8]

It is given that $z_1 = 3\,\text{cis}\left(\frac{3\pi}{4}\right)$ and $z_2 = 2\,\text{cis}\left(\frac{n\pi}{16}\right)$, $n \in \mathbb{Z}^+$.

(a) In parts (a)(i) and (a)(ii), give your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, $r \geq 0$, $-\pi < \theta \leq \pi$.

(i) Find the value of $z_1^{\,3}$.

(ii) Find the value of $\left(\frac{z_1}{z_2}\right)^4$ for $n = 2$. [5]

(b) Find the least value of $n$ such that $z_1 z_2 \in \mathbb{R}^+$. [3]

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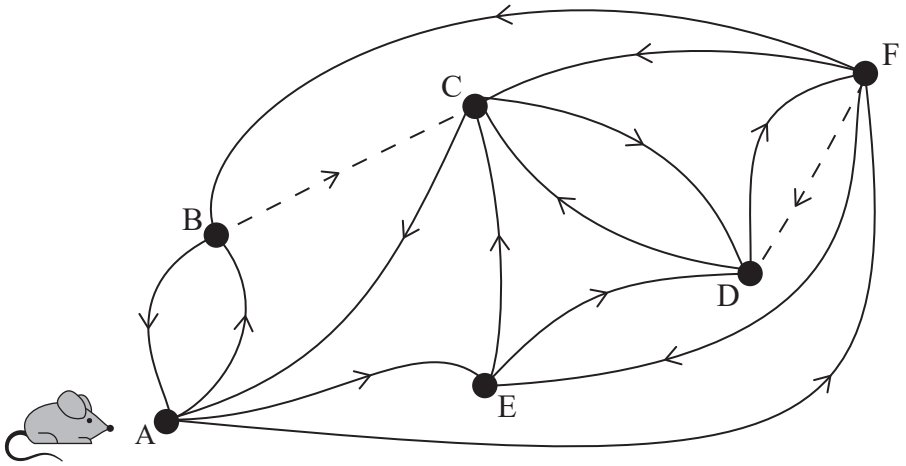
**13.** [Maximum mark: 8]

The graph below shows a small maze, in the form of a network of directed routes. The vertices $A$ to $F$ show junctions in the maze and the edges show the possible paths available from one vertex to another.

A mouse is placed at vertex $A$ and left to wander the maze freely. The routes shown by dashed lines indicate paths sprinkled with sugar.

When the mouse reaches any junction, she rests for a constant time before continuing.

At any junction, it may also be assumed that
- the mouse chooses any available normal path with equal probability
- if the junction includes a path sprinkled with sugar, the probability of choosing this path is twice that of a normal path.




(a) Determine the transition matrix for this graph. [3]

(b) If the mouse was left to wander indefinitely, use your graphic display calculator to estimate the percentage of time that the mouse would spend at point $F$. [3]

(c) Comment on your answer to part (b), referring to at least one limitation of the model. [2]

**(This question continues on the following page)**

**(Question 13 continued)**

**14.** [Maximum mark: 7]

A geometric transformation $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x' \\ y' \end{pmatrix}$ is defined by

$$T: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}.$$

(a) Find the coordinates of the image of the point $(6, -2)$. [2]

(b) Given that $T: \begin{pmatrix} p \\ q \end{pmatrix} \mapsto 2\begin{pmatrix} p \\ q \end{pmatrix}$, find the value of $p$ and the value of $q$. [3]

(c) A triangle $L$ with vertices lying on the $xy$ plane is transformed by $T$.

Explain why both $L$ and its image will have exactly the same area. [2]

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**15.** [Maximum mark: 7]

The number of coffees sold per hour at an independent coffee shop is modelled by a Poisson distribution with a mean of $22$ coffees per hour.

Sheila, the shop's owner wants to increase the number of coffees sold in the shop. She decides to offer a discount to customers who buy more than one coffee.

To test how successful this strategy is, Sheila records the number of coffees sold over a single $5$-hour period. Sheila decides to use a $5\%$ level of significance in her test.

(a) State the null and alternative hypotheses for the test. [1]

(b) Find the probability that Sheila will make a type I error in her test conclusion. [4]

Sheila finds $126$ coffees were sold during the $5$-hour period.

(c) State Sheila's conclusion to the test. Justify your answer. [2]

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**16.** [Maximum mark: 8]

A particle $P$ moves in a straight line, such that its displacement $x$ at time $t\ (t \geq 0)$ is defined by the differential equation $\dot{x} = x\cos t\left(\mathrm{e}^{-\sin t}\right)$. At time $t = 0$, $x = \frac{1}{\mathrm{e}}$.

(a) By using Euler's method with a step length of $0.1$, find an approximate value for $x$ when $t = 0.3$. [3]

(b) By solving the differential equation, find the percentage error in your approximation for $x$ when $t = 0.3$. [5]

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**References:**

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