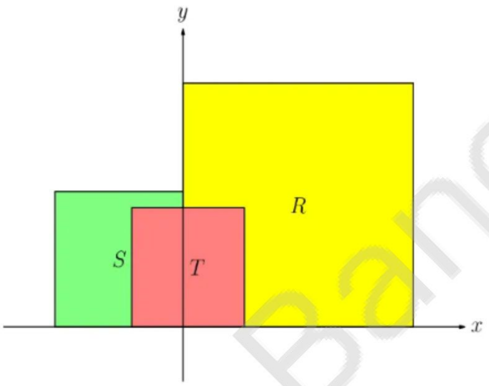
MATHEMATICAL RESEARCH ACADEMY

AMC 10A, 2022, Problem 25

Let R, S and T,be squares that have vertices at lattice points (i.e., points whose coordinates are both integers) in the coordinate plane, together with their interiors. The bottom edge of each square is on the x-axis.The left edge of R and the right edge of S are on the y-axis and R contains $\frac{9}{4}$ as many lattice points as does S .The top two vertices of T are in $R \cup S$,and T contains $\frac{1}{4}$ of the lattice points contained in $R \cup S$.See the figure (not drawn to scale).



The fraction of lattice points in S that are in $S \cap T$ is 27 imes the fraction of lattice points in R that are in $R \cap T$. What is the minimum possible value of the edge length of R plus the edge length of S plus the edge length of T ?

A) 336

B) 337

C) 338

D) 339

E) 340

AMC 10A, 2022, Problem 23

Isosceles trapezoid $ABCD$ has parallel sides $\overline{AD}$,and $\overline{BC}$,with $BC < AD$ and $AB = CD$.There is a point P in the plane such that $PA = 1, PB = 2, PC = 3$ and $PD = 4$ .What is $\frac{BC}{AD}$

A) $\frac{1}{4}$

B) $\frac{1}{3}$

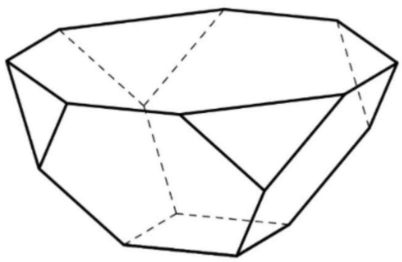
C) $\frac{1}{2}$

D) $\frac{2}{3}$

E) $\frac{3}{4}$

AMC 10A, 2022, Problem 21

A bowl is formed by attaching four regular hexagons of side 1 to a square of side 1 .The edges of the adjacent hexagons coincide, as shown in the figure. What is the area of the octagon obtained by joining the top eight vertices of the four hexagons, situated on the rim of the bowl?



A)6

B)7

C)$5+2\sqrt{2}$

D)8

E)E